Name: ____ Period: __________ Date: _______________

Geometric Sequences Notes

Definition:

A **geometric progression** is a sequence of numbers in which each term, after the first, is obtained by multiplying the preceding number by a constant called **common ratio**. The element of a sequence is called **terms**.

Notations for Geometric Sequence:

a = *first term* **r** = *common ratio*

n= *number of terms* $\mathbf{a_n}$= *nth term*

S = *sum of geometric sequence*

Formula of Geometric Sequence:

Terms of a geometric sequence may be represented by $a, ar, ar^2, ar^3, \dots, ar^{n-1}$ from the foregoing elements, the nth term, denoted by a_n, of a geometric progression is $a_n = ar^{n-1}$

The sum of the terms of geometric sequence S,

$$S = a + ar + ar^2 + \cdots + ar^{n-1}$$

and multiply each term by r, i.e.

$$S(r) = ar + ar^2 + \cdots + ar^{n-1} + ar^n$$

then

$$\begin{aligned} S &= a + ar + ar^2 + \cdots + ar^{n-1} \\ (-)S(r) &= \quad\;\; ar + ar^2 + \cdots + ar^{n-1} + ar^n \\ \hline S(1-r) &= a - ar^n \end{aligned}$$

Sum of the first **n** term a) $S = \frac{a-ar^n}{1-r}\ or\ S = \frac{a(1-r^n)}{1-r}$, r ≠ 1

b) $S = \frac{a-ar^n}{1-r} = \frac{a-r(ar^{n-1})}{1-r} = \frac{a-ra_n}{1-r}$, r ≠ 1

Infinite geometric sequence

if a is the first term and r is the common ratio of an infinite geometric sequence, and if $|r| < 1$, then the sum of the terms of the sequence is given by the formula

c) $S = \frac{a}{r-1}$, r ≠ 1

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Name: ____ Period: __________ Date: _______________

Geometric Sequences Notes

Sample Problem 1: The first three terms of a geometric sequence are given. Find the next three terms of the following geometric sequence.

1. 2, 4, 8,........

Common ratio: 2

Solution: $8(2) = 16, 16(2) = 32 \ and \ 32(2) = 64$

Answer: 16, 32 and 64

2. $-3, 1, -\frac{1}{3}, \dots ..$

Common ratio: $-\frac{1}{3}$

Solution:

$$\left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right) = \frac{1}{9},\ \left(\frac{1}{9}\right)\left(-\frac{1}{3}\right) = -\frac{1}{27}, \left(-\frac{1}{27}\right)\left(-\frac{1}{3}\right) = \frac{1}{81}$$

Answer: $\frac{1}{9}, -\frac{1}{27} \ and \ \frac{1}{81}$

Sample Problem 2: Solve the following problems involving the nth term of geometric sequences.

4. Find the 6th term in the Geometric progression 3, 6, 12,....

Given: $a_1 = 3;\quad r = \frac{12}{6} = 2$

Solution:

$$a_6 = 3(2)^{6-1} = 3(2)^5 = 3(32) = 96$$

Answer: 96

5. Find the eighth term in the geometric sequence 243, 81, 27,....

Given: $a_1 = 243; r = \frac{27}{81} = \frac{1}{3}$

Solution:

$$a_8 = 243\left(\frac{1}{3}\right)^{8-1} = 243\left(\frac{1}{3}\right)^7 = 243\left(\frac{1}{2187}\right) or \ \frac{3^5}{3^7} = \frac{1}{3^2} = \frac{1}{9}$$

Answer: $\frac{1}{9}$

Sample Problem 3: Find the fifth term of the following geometric sequence given their first term and the common ratio.

5. a = 3; r= 2

Solution:

$$a_5 = (3)(2)^{5-1} = 3(2)^4 = 3(16) = 48$$

Answer: 48

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Name: ____ Period: ___________ Date: _________________

Geometric Sequences Notes

6. a= 5; r = -1

Solution:

$a_5 = (5)(-1)^{5-1} = 5(-1)^4 = 5(1) = 5$

Answer: 5

Sample Problem 4: Solve problem involving the sum of geometric sequence.

7. Solve for the sum of the first 6 term of a geometric sequence -1, -5, 25,....
Given: a =-1; r= 5; n = 6
Solution:

$$S = (-1)\frac{(1-(5)^6)}{(1-5)} = (-1)\frac{(1-15625)}{-4} = (-1)\frac{-15624}{-4} = \frac{15624}{-4} = -3906$$

8. The third term of a geometric sequence is 20 and the fifth term is 80 what is the second term?

Equation:

$20 = ar^{3-1} = ar^2; 80 = ar^{5-1} = ar^4$

Solution:

Step 1:
$\frac{80}{20} = \frac{ar^4}{ar^2}$; $4 = r^2$; $\sqrt{4} = \sqrt{r^2}$; $r = 2$
Step 2:

$20 = ar^2$; $20 = a(2)^2$; $20 = 4a$; $a = 5$

Step 3:

$a_2 = (5)(2) = 10$

9 The population of an island increases by 10% each year. If the initial population is 500, what is the expected population after 5 years?
Given: a= 500, r =10% or 0.1= (1+0.1)=1.1; n = 5
Solution:

$$S = (500)\frac{(1-(1.1)^5)}{(1-1.1)} = (500)\frac{(1-1.61051)}{-0.1} = (500)\frac{-0.61051}{-0.1} = \frac{-305.255}{-0.1} = 3052.55$$

The expected population after 5 years is 3052

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Name: ____ Period: __________ Date: _______________

Geometric Sequences Notes

10. A certain type of bacteria doubles in number every day. If an initial amount of 1,000 bacteria was counted, what is the population after 5 days?

Given: a= 1000; r = 2; n = 5

Solution:

$$S = (1000)\frac{(1-(2)^5)}{(1-2)} = (1000)\frac{(1-32)}{-1} = (1000)\frac{-31}{-1} = \frac{-31000}{-1} = 31000$$

The expected number of bacteria after 5 days is 31000

Sample Problem 5: Find the sum of an infinite geometric sequence

11. Find the sum of the terms of the infinite geometric sequence 125, 25, 5,…..

Given: $a = 125; r = \frac{1}{5}$

Solution:

$$S = \frac{a}{1-r} = \frac{125}{\left(1-\frac{1}{5}\right)} = \frac{125}{\frac{4}{5}} = \frac{5}{4}(125) = \frac{625}{4}$$

12. Find the sum of the infinite geometric sequence 64,-4,1/4,…

Given: $a = 64; r = -\frac{1}{16}$

Solution:

$$S = \frac{a}{1-r} = \frac{64}{\left(1-\left(-\frac{1}{16}\right)\right)} = \frac{64}{\frac{17}{16}} = \frac{16}{17}(64) = \frac{1024}{17}$$